

Using Simulation for Locating Transmitter in a Multistatic Sensor Network

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Abstract— Multistatic sensor networks consist of independent transmitters (sources) and receivers with a shared area coverage. Some important applications of bi/multistatic sensors in space systems include the use of spatially separated receiver and transmitter satellites, design of space radars and spaceborn bi/multistatic synthetic aperture radar. Sensing zone of a single transmitter-receiver couple in such systems is defined by Cassini ovals, a family of quartic plane curves with unique properties. Therefore, the problem of locating sensors in multistatic settings is a challenge for decision-makers. In this study, considering a given set of receiver locations, we aim to determine the location for a single transmitter to maximize the total area covered. To achieve this we carry out exhaustive Monte Carlo simulations which measure the coverage achieved for all possible transmitter location alternatives for a given receiver field. We experiment with both randomly and polygonally deployed receiver locations and present results for a number of selected examples. Our work reveals that the coverage can significantly be improved by selecting appropriate transmitter locations and simulation is a useful tool to design multistatic fields.

Keywords— *multistatic, Cassini oval, location analysis, simulation*

I. INTRODUCTION

Radar and sonar networks are mainly composed of transmitter and receivers, and the coverage achieved by those networks is of great importance to decision-makers conducting search in a surveillance area [1]. Basic operating concept of a radar/sonar is to emit sound energy from the transmitter to the environment and listen for the reflected echoes returning across the receivers to detect, localize and track targets of interest [2].

A radar/sonar system is called a *monostatic* sensor if the transmitter (source) and receiver are co-located. A *bistatic* system is a generalization of the traditional monostatic sensor to the case where the transmitter and receiver are not co-located. A multistatic sensor network consists of multiple transmitters and receivers – each transmitter and receiver couple forms a bistatic system - distributed over the surveillance area [3, 4].

Bi and multistatic systems have a broad range of applications areas both in space and underwater. Some important applications of bi/multistatic sensors in space systems include the use of spatially separated receiver and transmitter satellites, design of space radars and spaceborn bi/multistatic synthetic aperture radar (SAR). Fig. 1 illustrates an example spaceborn multistatic radar system with one transmitter and two receivers. Bistatic space radars received increasing attention with respect to SAR applications and several spaceborne bi and multistatic radar missions have been proposed by many researchers.

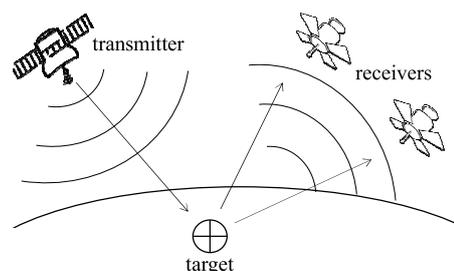


Fig. 1. Spaceborn multistatic radar system with one transmitter and two receivers

Spaceborne SAR systems that include multiple antenna beams and/or multiple sensors on board of multiple platforms flying on different orbits have a number of advantages over traditional monostatic systems. In those systems multiple radar signals received from different spatial positions are processed jointly to enable improved detection and coverage probabilities in the surveillance field.

The most important advantages of bi and multistatic sensor in spaceborn radars include enhanced coverage of stationary or mobile targets on the earth surface, improved imaging capability, protection against interfering targets (robustness to sensor loss and jamming) [5]. Other advantages include, expanded geometric diversity (greater coverage footprint), increased target hold, improved localization through cross-fixing [6, 7]. In military applications, it also makes it more difficult for the contact (target) to be able to pinpoint the location of the searcher. In the underwater setting, it also allows multi-platform operations such as airplane deployed receivers with surface ship sources [8]. The main disadvantage that multistatics is the increased system complexity and unusual sensing zone geometries determined by the transmission losses.

In particular, the sensing zone of a bistatic couple is described by Cassini ovals. These ovals take various shapes depending on the sensor geometry and transmitter strength/receiver gain power. For this reason, the problem of determining multistatic sensor locations is harder than the problem for traditional monostatic systems where the coverage of a sensor is modeled by regular disks. In order to take advantage of multistatic networks, optimal sensor location schemes should be devised.

In this study, we tackle a basic location problem for those systems which include multiple receivers and a single transmitter. In particular, we consider a number of stationary receivers located in a field of interest. Given receiver locations, we seek to determine the best transmitter location which yields the highest area coverage. For this purpose, we

first discretize the search area into small rectangular grids and carry out exhaustive Monte Carlo simulations to measure the coverage achieved for each possible transmitter location alternative. To compute the coverage performance, we adopt a binary coverage model, i.e. full coverage within the sensing zone modelled by a Cassini oval, no coverage outside. The binary coverage modelled is sometimes called as the definite-range or cookie-cutter coverage model. In these paper, we experiment with both randomly and polygonally deployed receiver locations and present results for a number of selected examples.

In all experiments we assumed that transmitters and receivers are stationary, i.e. there is no allowance for the effect of motion. Besides, we ignore several practical issues such as tracking performance, direct blast effect, and energy management. Having a planar assumption, multipath propagation and varying sensor depths (for underwater setting) are neglected. The environment is assumed to be homogeneous; therefore, the detection range is constant throughout the search region.

The organization of the paper is as follows. We provide a brief literature review of bistatic and multistatic sensor work in Section II. We describe the preliminaries for the bistatic detection theory and Cassini oval geometry in Section III. Section IV presents the details of our simulation methodology and examples of experiments. Finally, we conclude in Section V.

II. RELATED WORK

The ongoing multistatic work in the literature can be categorized as the sensor & ping optimization studies and multistatic tracking algorithms studies. In their study [9] DelBalzo, et al. analyzed the oceanographic effects on multistatic sonobuoy fields and present near optimal solutions for locating sensors in heterogeneous environments. In [10] Walsh and Wettergren approximated the search performance for a multistatic field and compute the expected probability of a successful search operation for a given target track. Their solution approach depends on parameters such as the numbers of transmitters and receivers in the search field, their probability distributions of sensor locations, and the location and orientation of the target track. In another study, [8] proposed an algorithm to improve the performance of multistatic systems which involve mobile transmitters. The algorithm decides the path of the transmitters and select a subset from the available passive sensors (receivers) with the objective of maximizing tracking performance. In [11], the authors studied the problem of determining optimal placement of multistatic sensors to achieve maximal coverage while minimizing the number of sensors required.

In [12] the problem of determining effective pinging strategies in multistatic sonar systems with multiple transmitters is addressed. For maintaining an effective coverage performance, a probability of target presence metric formulation is used. This formulation utilizes sonar performance prediction and a Bayesian update to incorporate negative information (i.e., searching an area but finding no targets) into an optimization procedure. In [13] the effects of uncertainty on the distribution of likely outcomes of multistatic fields are modeled by a combination of Monte Carlo simulation methods and Bayesian fusion techniques.

The problem of dynamic optimization of ping schedule in an active sonar buoy network deployed to provide persistent surveillance of a littoral area through multistatic detection is studied in [14]. In [15], Wang, et al. formulated the sensor scheduling problem for multistatic active sonar sensor networks and discuss the algorithms that schedule sensors to achieve desired duty cycles, while optimizing both the temporal and spatial sensing coverage. In [7], the authors investigated the problem of determining optimal sensor placements for maximizing information provided to the multistatic trackers. An approximate performance model for undersea surveillance using multistatic sensor fields is developed in [10]. The model provides an upper bound on the field-level detection performance that is achievable by systems comprised of separate sources and receivers.

The Multistatic Performance Prediction Methodology (MPPM) developed in [16] can be used to evaluate detection as a function of source and receiver densities. MPPM is based on evaluating the sonar equation, probabilistic assumptions. It lacks explicit treatment of target actions, is not suited to evaluate tactics. In [17-29] tracking algorithms for multistatic systems are analyzed and it is considered that the system sensors make decisions and transmit these results to the fusion center for track estimate. The tracker performance models provided in those studies are simple tools that predict tracker performance as a function of scenario parameters that include assumptions about target dynamics, sensor performance, and tracker settings. Tracker performance metrics include detection performance as measured through tracker receiver operating curves, as well as localization accuracy.

In terms of area coverage performance of bistatic and multistatic sensors, [2] and [30] develop analytic formulas that approximate the coverage performance of randomly deployed multistatic sensor fields and sweep width for mobile transmitters. In [31] authors develop optimal bistatic sensor location schemes for maximizing the detection probability of stationary targets. The direct blast effect of multistatic sensors is assessed in [32]. In this study authors employ Monte Carlo simulation and measure the coverage loss resulting from the direct blast effect with respect to the number of transmitters and receivers. [33-36] are some of the recent studies that approach the multistatic sensor location problem from a point coverage perspective. Different from the area coverage problems, a point coverage model aims to position sensors in such a way as to cover as many of a number of fixed point (target) locations as possible.

III. PRELIMINARIES

A. Bistatic Detection

In a sonar sensor network detection happens if the sound energy emitted by the transmitter that is reflected off the target generates an energy that is at least as much as the energy threshold of one or more receivers. This energy threshold, TH , depends on the sensitivity of the receiver, environmental conditions and detection and false alarm settings. Then, for a bistatic system detection happens if:

$$SL - TL_1 - TL_2 \geq TH \quad (1)$$

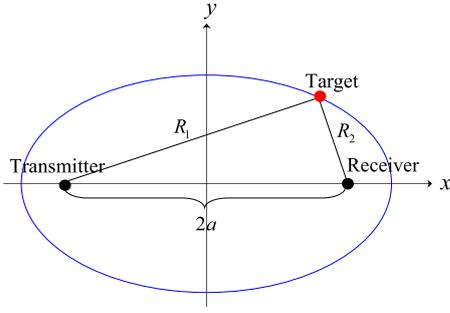


Fig. 2. Cassini oval with foci F_1 and F_2 on the x -axis.

where SL is the source level, TL_1 and TL_2 are the transmission loss values from transmitter to target and target to receiver, respectively. If we further assume that the environment is homogeneous such that the transmission loss follows a simple power law then for some constant m we can rewrite (1) as:

$$SL - m \log(R_1) - m \log(R_2) \geq TH \quad (2)$$

where R_1 and R_2 denote the ranges from transmitter to target and target to receiver, respectively. Solving this equation for the bistatic range product, detection will occur if this product is smaller than some threshold value b^2 :

$$R_1 R_2 \leq 10^{\frac{1}{m}(SL-TH)} \equiv b^2 \quad (3)$$

In (3), b is defined as the equivalent monostatic detection range or the geometric mean range of the system that represents the performance of a bistatic system when source and receiver are co-located [38]. Equation (3) is also the inequality that defines the interior of a Cassini oval. So, from now on, we will use Cassini ovals to model the shape of a sensing zone for bistatic sensors.

B. Cassini Ovals

These curves are characterized in such a way that the product of the distances from two fixed focal points F_1 and F_2 is constant b^2 and the distance $(F_1, F_2) = 2a$ [39]. Therefore it is possible to express a Cassini oval by using the parameters a and b where a is the semi-distance between the two foci and b is the constant which determines the exact shape of the curve as will be discussed later. Cassini ovals are defined in two-center bipolar coordinates by the equation below:

$$d_1 d_2 = b^2 \quad (4)$$

where d_1 and d_2 denote the ranges from F_1 to P and P to F_2 , respectively as in Fig. 2.

If we locate the origin $(0,0)$ of Cartesian plane at the midpoint of the two foci F_1 and F_2 , and choose the x -axis as the line joining them, then the foci will have the coordinates $(\pm a, 0)$. The Cassini shape depends on parameters a and b , and form of the ovals can be characterized by evaluating the ratio a/b as in Table I.

Fig. 3 shows a family of Cassini ovals where the parameter b is fixed and the ovals are drawn for several different values of a . Interested reader can also refer to [37] for more details on the properties of these ovals and their application areas.

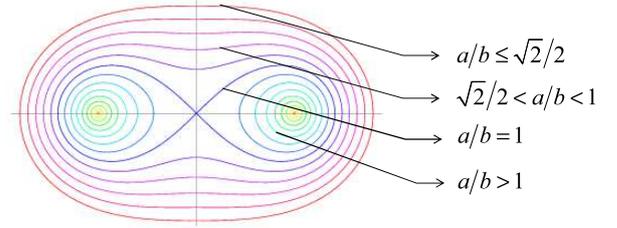


Fig. 3. A family of Cassini Ovals (a fixed)

TABLE I. CASSINI OVAL SHAPES

a/b Ratio	Oval Shape
$a/b \leq \sqrt{2}/2$	the curve is a single loop that looks like an ellipse
$\sqrt{2}/2 < a/b < 1$	the oval attains a dent on top and bottom
$a/b = 1$	the curve is a Lemniscate of Bernoulli
$a/b > 1$	the curve splits into two mirror-imaged disjoint ovals

IV. SIMULATION EXPERIMENTS

In our simulation setting, we consider a surveillance region A and a sub-region A' immersed in the center of A . Without loss of generality, we assume both A and A' are square with lengths m and $m' = m - 2b$, respectively. Assuming x receivers are already placed inside A' , we wish to determine the best location for the only transmitter available such that the total area covered is maximized. Note that the total sensing zone of a single transmitter and multiple receiver network can be described as a collection of overlapping Cassini ovals. [38] contains an approximation for calculating the area of a single Cassini oval using parameters a and b . However, the existence of multiple and overlapping ovals makes that approximation useless. For this reason, we employ numerical simulation to compute the effective area coverage (EAC) of a multistatic network. The EAC is simply the area covered at least by one bistatic sensor pair. The difficulty in computing EAC for a given set of transmitter and receiver locations is handled by discretizing the area A (and A') into relatively small square grids and approximating EAC by simply counting the number of grids that lie inside the sensing zone of the network.

Adopting a fixed value of b , the criteria for deciding whether a grid is covered or not is based on the bistatic detection formula given in (4). For a specific receiver field layout, the best transmitter location is determined by selecting the transmitter location among all alternatives which yield the highest EAC. Fig. 4 shows an example search region which includes two receivers and a single transmitter. For each transmitter-receiver pair, there exists a Cassini oval shaped sensing zone. Each grid (little squares) represents a possible transmitter location and, using the binary coverage model, a grid is assumed to be covered if its center is located inside one of the ovals.

Suppose $r \in R$ and $g \in G$ denote the set of receivers and grids, respectively. Let (x_r, y_r) and (x_g, y_g) denote the locations of receiver r and grid g . Let (x'_g, y'_g) denote the location of the grid the transmitter is located at.

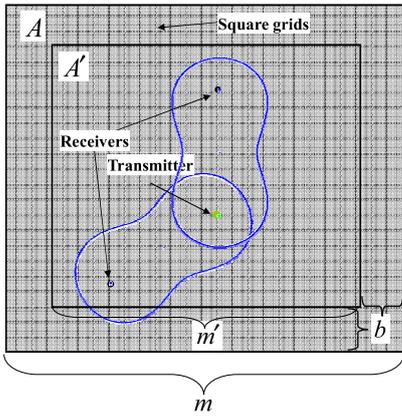


Fig. 4. Discretized search area A .

Using (4), for a given transmitter location (x_g^t, y_g^t) , a grid g is said to be completely covered if the inequality
$$\sqrt{[(x_g - x_r)^2 + (y_g - y_r)^2]} \leq b$$
 holds for at least one receiver $r \in R$. For a given receiver deployment layout, above mentioned computation is repeated for all possible locations of transmitter $(x_g^t, y_g^t), \forall g \in G$, receivers $(x_r, y_r), \forall r \in R$ and grids $(x_g, y_g), \forall g \in G$. At each iteration, let n_g denote the total number of grids covered for transmitter located at grid g . Then the EAC achieved for transmitter location (x_g^t, y_g^t) is simply calculated as:

$$\text{EAC}_g = \frac{n_g}{|G|} \cdot A \quad (5)$$

We now present examples of our experiment results for fields with different receiver deployment layouts and receiver numbers. Fig.5 represents examples of randomly deployed receiver fields for $|R|=3, 4$, and 8. For each case, the figure shows (left) resulting EAC for all transmitter location alternatives represented by a color map (hot colors represent preferable grids and cold colors the opposite) and (right) actual receiver locations and the optimal transmitter location. In this group of examples, the receivers are dispersed inside the area randomly, i.e. there is no control over receiver locations. In such scenarios it is often hard to predict the best transmitter location since the overall coverage is represented by a set of overlapping Cassini ovals of different shapes and sizes.

Additional experiments are carried out to analyze the polygonal shaped receiver fields as demonstrated in Fig. 6. In these runs receivers are deployed around the center of the region A' with equal distances and angles. These examples reveal that, in polygonal receiver regions, it is often best to place the source in the center of the receivers. However, there are examples where this tactic does not always yield the highest coverage. In the case displayed Fig 6(b), one can get a higher coverage if the transmitter is placed closer towards any of the two receivers rather than placing it directly at the center of region A . We also note that, the overall coverage is affected significantly by the transmitter-receiver distances as can be seen in Fig. 6(c) and (d). As the receivers are located far apart from one another such that $a/b > 1$, the sensing zones become two disjoint ovals, and yield decreased EAC.

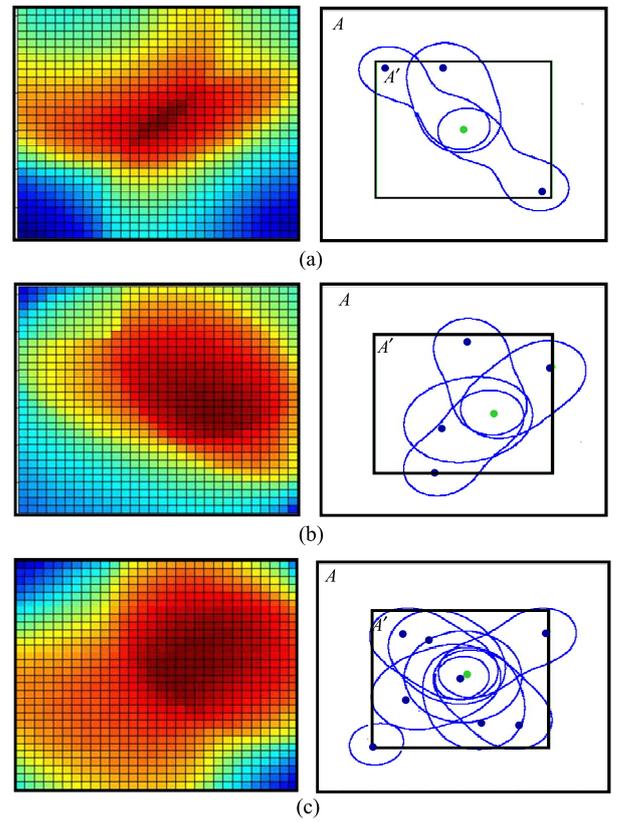


Fig. 5. Examples for (a) $|R|=3$, (b) $|R|=4$, and (c) $|R|=8$ randomly deployed receivers

V. CONCLUSION

In this study, considering a multistatic setting, we focus on the problem of determining the optimal location for a single transmitter inside a field which includes a set of receivers. Since the sensing zone of each transmitter-receiver pair is modeled with a family of Cassini ovals, it is not possible to calculate the overall coverage of overlapping multiple ovals with analytic models. For this reason, we carried out a number of Monte Carlo simulations which approximate the coverage for each possible transmitter location.

Our results showed that, for randomly deployed receiver fields, it is a difficult task to predict the optimal transmitter location because of the irregular coverage profiles and unexpected overlaps for each source-receiver couple. The second group of experiments showed that, for polygonal shapes, the optimal transmitter locations are not always necessarily at the center of the receivers.

These initial results and optimal multistatic geometries can be used as a guideline by designers both to select important system characteristics, i.e., source strength, receiver gain, and to plan the specific deployment patterns of multistatic fields both in spaceborn or underwater applications. The planners should always keep in mind that the coverage performance of a multistatic system strongly depends on the transmitter-receiver geometry and the distances among all sensors.

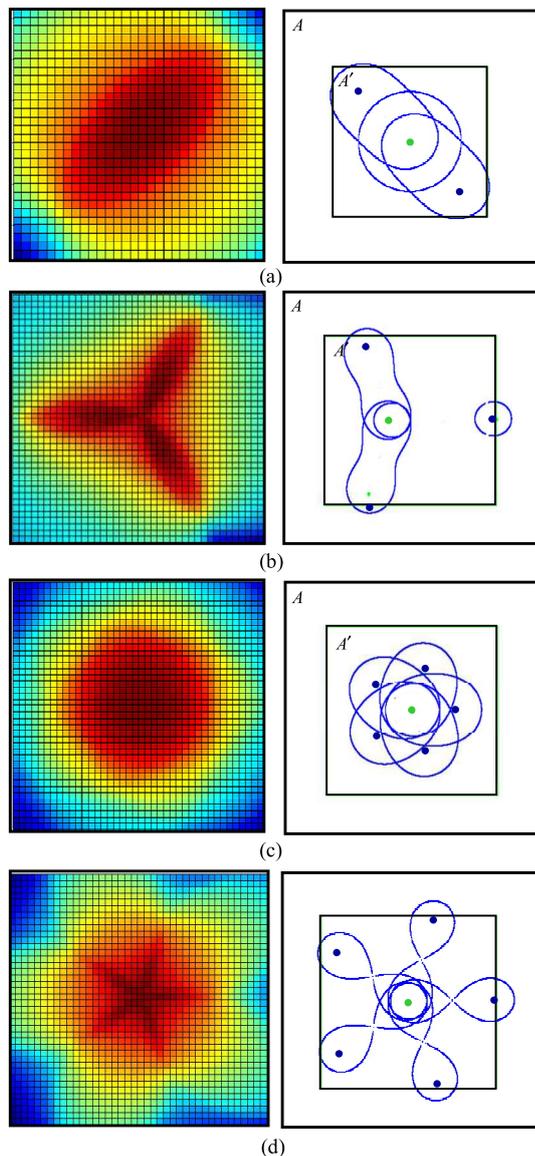


Fig. 6. Examples for (a) $|R|=2$, (b) $|R|=3$, (c) $|R|=5$, and (d) $|R|=5$ polygonally deployed receivers

Although the Monte Carlo simulation approach presented in this study is limited to a single transmitter case, it can be applied to multiple transmitter location problems. A key challenge in those problems would be the inclusion of scheduling and coordination issues of the whole sensor network as well as the computational requirement. The optimization of multistatic fields under heterogeneous environments where the parameter b differs at different parts of the region is another possible future work.

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